Class Exercise 1 Solution

1. Evaluate the iterated integral
$$\int_0^2 \int_0^1 \frac{x}{1+xy}\, dxdy\ .$$

Solution. Let $R = [0,1] \times [0,2]$. By Fubini's theorem,
$$\begin{aligned}
\int_0^2 \int_0^1 \frac{x}{1+xy}\, dxdy &= \iint_R \frac{x}{1+xy}\, dA \\
&= \int_0^1 \int_0^2 \frac{x}{1+xy}\, dydx \\
&= \int_0^1 \log(1+xy)\Big|_{y=0}^{y=2} dx \\
&= \int_0^1 \log(1+2x)\, dx \\
&= \frac{1}{2}[(1+2x)\log(1+2x) - 2x]\Big|_{x=0}^{x=1} \\
&= \frac{3}{2}\log 3 - 1\ .
\end{aligned}$$

Here log is the natural log.

2. Let f be a function defined in the square $R = [0,1] \times [0,1]$, $f(x,y) = -1$ for $x < y$ and $f(x,y) = 5$ for $x \geq y$. Evaluate
$$\iint_R f(x,y)\, dA\ .$$

Solution. By Fubini's,
$$\begin{aligned}
\iint_R f(x,y)\, dA &= \int_0^1 \int_0^1 f(x,y)\, dydx \\
&= \int_0^1 \int_0^x f(x,y)\, dydx + \int_0^1 \int_x^1 f(x,y)\, dydx \\
&= \int_0^1 \int_0^x -1\, dydx + \int_0^1 \int_x^1 5\, dydx \\
&= \int_0^1 (-x)\, dx + \int_0^1 5(1-x)\, dx \\
&= -\frac{1}{2} + \frac{5}{2} \\
&= 2\ .
\end{aligned}$$